The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 7, 2010 Course: EE 313 Evans

Name: Set, Solution
Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and homework solution sets.
- **Power off all cell phones**
- You may use any standalone calculator or other computing system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise.**

| Problem | Point Value | Your score | Topic |
|---|---|---|---|
| 1 | 18 | | Differential Equation |
| 2 | 18 | | Convolution |
| 3 | 24 | | System Properties |
| 4 | 28 | | Equalization |
| 5 | 12 | | Potpourri |
| Total | 100 | | |

**Problem 1.1** Differential Equation. *18 points.*

For a continuous-time system with input $x(t)$ and output $y(t)$ governed by the differential equation

$$\frac{d^2}{dt^2}y(t)+5\frac{d}{dt}y(t)+6y(t)=x(t)$$

for $t \geq 0^+$.

(a) What are the characteristic roots of the differential equation? *3 points.*

$$\lambda^2+5\lambda+6=0$$

$$(\lambda+2)(\lambda+3)=0$$

Characteristic roots are $\lambda=-2$ and $\lambda=-3$

(b) Find the zero-input response assuming non-zero initial conditions. Please leave your answer in terms of $C_1$ and $C_2$. *6 points.*

$$y_0(t) = C_1 e^{-2t} + C_2 e^{-3t}$$

(c) Find the zero-input response for the following initial conditions: $y(0^+) = 0$ and $y'(0^+) = 1$. *6 points.*

$$y_0'(t) = -2C_1 e^{-2t} - 3C_2 e^{-3t}$$

$$y_0(0^+) = C_1 + C_2 = 0 \Rightarrow C_1 = -C_2$$

$$y_0'(0^+) = -2C_1 - 3C_2 = 1 \Rightarrow -C_2 = 1 \Rightarrow C_2 = -1$$

$$y_0(t) = e^{-2t} - e^{-3t}$$

(d) Is the zero-input response asymptotically stable, marginally stable, or unstable? Why? *3 points.*

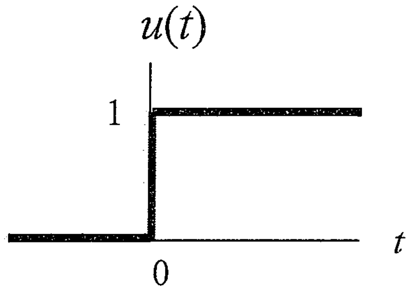
Both characteristic roots have negative real values.

The characteristic modes die out as $t \to \infty$:

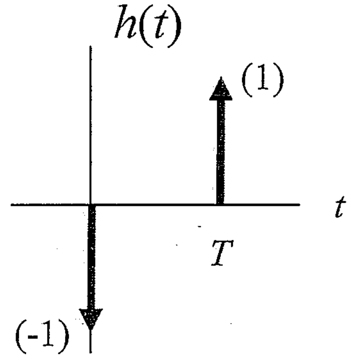
$\lim_{t\to\infty} e^{-2t} = 0$ and $\lim_{t\to\infty} e^{-3t} = 0$. Asymptotically stable.

**Problem 1.2** Convolution. *18 points.*

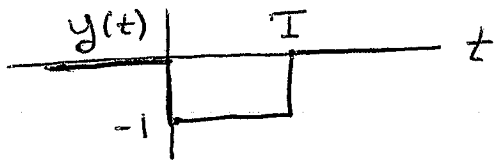
Sketch (plot) the following convolutions. On the sketches, be sure to label significant points on the horizontal and vertical axes. You do not have to show intermediate work, but showing intermediate work may qualify for partial credit.



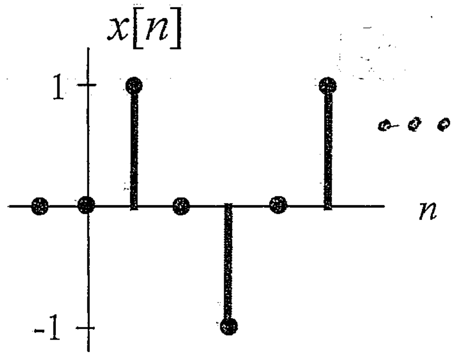

(a) In continuous-time, convolve the unit step function $u(t)$ with $h(t)$ where $h(t) = -\delta(t) + \delta(t - T)$. Both signals are plotted on the right. *9 points*

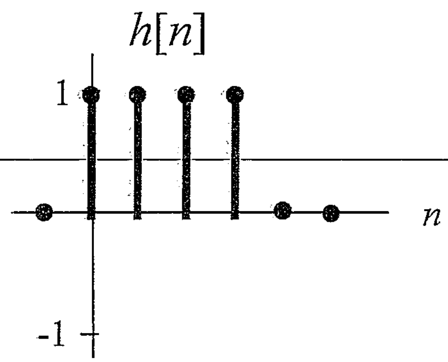



$$y(t) = u(t) * h(t)$$
$$= u(t) * (-\delta(t) + \delta(t-T))$$
$$= -u(t) + u(t-T)$$



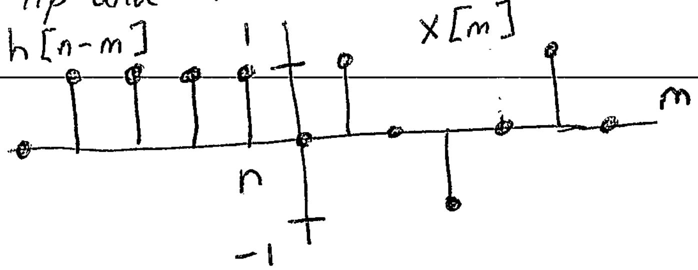

(b) In discrete time, convolve a one-sided sine wave $x[n]$ where $x[n] = \sin(\frac{\pi}{2} n)\, u[n]$ with a rectangular pulse of four samples in duration $h[n]$. Both signals are plotted on the right for $n \in [-1, 5]$. *9 points*

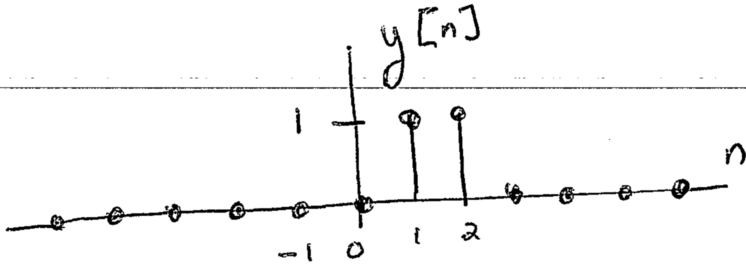



$$y[n] = x[n] * h[n]$$
$$= \underbrace{\sin\left(\frac{\pi}{2} n\right) u[n]}_{\text{one-sided}} * h[n]$$

Flip and slide $h[n]$ about $x[n]$

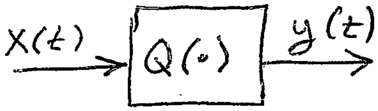


**Problem 1.3** System Properties. *24 points.*

Consider a continuous-time quantizer with input $x(t)$ and output $y(t)$ where

$$y(t) = \begin{cases} +1 & \text{if } x(t) \geq 0 \\ -1 & \text{otherwise} \end{cases}$$




The quantizer is a pointwise operation; that is, the current output value depends only on the current input value.

Either prove each of the following statements to be true, or give a counterexample to show that the statement is false. *Please note that writing only true or false will receive zero points.*

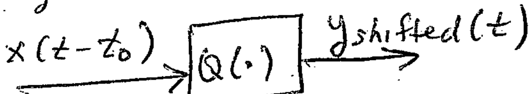
(a) The system is linear. *6 points.* False.

A system is linear if it is both homogeneous and additive. A necessary condition for homogeneity is that an all-zero input, i.e. $x(t) = 0$, would produce an all-zero output, i.e. $y(t) = 0$. However, when $x(t) = 0$, $y(t) = 1$. System is not linear.

(b) The system is time-invariant. *6 points.* True.

All pointwise systems are time-invariant.

Alternately:




$$y_{shifted}(t) = \begin{cases} +1 & \text{if } x(t-t_0) \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

Since $y_{shifted}(t) = y(t-t_0)$, system is time-invariant.

(c) The system is causal. *6 points.* True.

All pointwise systems are causal. (See p. 141 of Roberts' book.)

(All pointwise systems are also anti-causal.)

Causality means that the system does not rely on future input values or output values to compute the current output.

(d) The system is memoryless. *6 points.* True.

All pointwise systems are memoryless.

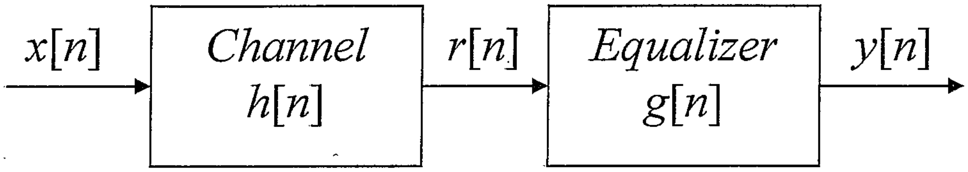
(It is possible that an implementation of a pointwise system is not memoryless.)

See p. 142 of Roberts' book, esp. highlighted section.

**Problem 1.4** Equalization. *28 points.*

In a communication system, the receiver uses equalization to compensate for distortion that a transmitted signal experiences when propagating through the communication channel.

Consider the following discrete-time model of a communication system:




$g[n]$ is the impulse response of a linear time-invariant equalizer,
$h[n]$ is the impulse response of a linear time-invariant communication channel,
$r[n]$ is the received signal,
$x[n]$ is the transmitted signal, and
$y[n]$ is the received signal.

Note: Two linear time-invariant systems in cascade. Order of cascade can be switched under assumption of exact precision computation.

The equalizer should be designed so that $h[n] * g[n] = \delta[n]$ in order to make $y[n] = x[n]$.

(a) Let $h[n] = (½)^n\, u[n]$ and $g[n] = g_0\, \delta[n] + g_1\, \delta[n\text{-}1]$. What are the values of $g_0$ and $g_1$? *14 points.*

$$h[n] * g[n] = \left(\tfrac{1}{2}\right)^n u[n] * \left(g_0\, \delta[n] + g_1\, \delta[n-1]\right)$$
$$= \left(\tfrac{1}{2}\right)^n u[n] * g_0\, \delta[n] + \left(\tfrac{1}{2}\right)^n u[n] * g_1\, \delta[n-1]$$
$$= g_0 \left(\tfrac{1}{2}\right)^n u[n] + g_1 \left(\tfrac{1}{2}\right)^{n-1} u[n-1] = \delta[n]$$

$n=0$: $\quad g_0 = 1$

$n=1$: $\quad g_0 \cdot \tfrac{1}{2} + g_1 = 0 \Rightarrow g_1 = -\tfrac{1}{2}$

(b) Let $h[n] = h_0\, \delta[n] + h_1\, \delta[n\text{-}1]$. Give a formula for $g[n]$ in terms of $h_0$ and $h_1$. *14 points.*

$$h[n] * g[n] = \left(h_0\, \delta[n] + h_1\, \delta[n-1]\right) * g[n]$$
$$= h_0\, \delta[n] * g[n] + h_1\, \delta[n-1] * g[n]$$
$$= h_0\, g[n] + h_1\, g[n-1] = \delta[n]$$

$$g[n] = \frac{1}{h_0}\, \delta[n] - \frac{h_1}{h_0}\, g[n-1]$$

First-order difference equation with characteristic root $-\frac{h_1}{h_0}$:

$$g[n] = \frac{1}{h_0}\left(-\frac{h_1}{h_0}\right)^n u[n]$$

Note: $h_0 = 1$ and $h_1 = -\frac{1}{2}$, $g[n] = \left(\frac{1}{2}\right)^n u[n]$.

**Problem 1.5** Potpourri. *12 points.*

Give one signal processing or communication system that uses each of the following subsystems and describe the role that the subsystem plays in the overall system. You can answer the question in either continuous time or discrete time.

(a) Linear time-invariant subsystem that is bounded-input bounded-output stable. *6 points.*

| Subsystem | System | Reference |
|---|---|---|
| 1. Lowpass RC filter | Switching – lowpass RC filter to debounce switch | Lecture |
| 2. Discrete-time lowpass filter | Blur an image (reduce high-frequency noise) | Mandrill demo |
| 3. Discrete-time highpass filter | Extract edges and texture from an image | Mandrill demo |
| 4. Lowpass filter | Analog-to-digital conversion | Slide 3-19 |
| 5. Resonator | Telephone touchtone detection | Reader K-20 |

(b) Linear time-invariant subsystem that is bounded-input bounded-output unstable. *6 points.*

| Subsystem | System | Reference |
|---|---|---|
| 1. Integrator | Analog-to-digital conversion (sigma-delta) | Reader K-35 |
| 2. Discrete-time integrator | Digital-to-analog conversion (sigma-delta) | |
| 3. Balance computation | Bank account | Roberts p. 138 |
| | | Slide 3-10 |
| 4. Integrator | Frequency modulation | Slide 3-9 |
| 5. Oscillator | Amplitude modulation | |
| 6. Oscillator | Digital communication system | Roberts Chapter 2, Problem 54 |
| 7. Resonator | Alarm system to detect an output of the resonator as its response grows without bound | Lecture |

Note: A resonator could be BIBO unstable (e.g. $h[n] = u[n]$) or BIBO stable (e.g. $h[n] = 0.9^n u[n]$).